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Strongly Implicit Procedure Applied to the Flowfield of Transonic Turbine Cascades

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Abstract

THIS synoptic presents an extension of a method for solving the potential equation that works well for isolated airfoils in the case of the transonic turbine cascade. Normally, the equation is solved by some relaxation method in a transformed plane that allows grid lines to be conformed to the blade surface. Ives and Liutermoza,[1] for example, use a conformal transformation and solve a quasilinear form of the nonconservative equation by an SLOR procedure. Ecer and Akay[2] use a finite-element formulation on a sheared Cartesian grid, which is easier to generate and avoids the singularity of ∞ encountered in a conformal mapping. For isolated airfoils, several factorized methods, such as the ADI, AF, and SIP have been shown to generally require less computer time and expenditure than the relaxation schemes SOR, SLOR, and several workers[3-6] have used these schemes for transonic flow problems. Recently, Sankar and Tassa[7] have shown the SIP to be competitive, if not superior, to the AF and ADI methods, particularly when the coefficients of the equation have wide spatial variations, as occurs on a highly stretched grid. We have thus chosen the SIP for the current investigation.

Contents

For compressible, two-dimensional, irrotational flow problems, the continuity equation, written in terms of an arbitrary coordinate transformation is

$$\frac{\partial}{\partial \xi}\left[\rho\left(A_1 \frac{\partial \phi}{\partial \xi}+A_2 \frac{\partial \phi}{\partial \eta}\right)\right]+\frac{\partial}{\partial \eta}\left[\rho\left(A_2 \frac{\partial \phi}{\partial \xi}+A_3 \frac{\partial \phi}{\partial \eta}\right)\right]=0 \tag{1}$$

where A_1, A_2, and A_3 are combinations of transformation metrics given by

$$A_1=(\xi_x^2+\xi_y^2)/J, \qquad A_2=(\xi_x\eta_x+\xi_y\eta_y)/J$$

$$A_3=(\eta_x^2+\eta_y^2)/J, \qquad J=\xi_x\eta_y-\xi_y\eta_x$$

where ξ and η are the transformed plane coordinates. The density ρ is related to the velocity potential through the isentropic energy equation

$$\rho=[1+\tfrac{1}{2}(\gamma-1)M^2(1-\phi_x^2-\phi_y^2)]^{1/(\gamma-1)} \tag{2}$$

and has been nondimensionalized with the inflow density.

Equations (1) and (2) are not amenable to efficient implicit solution, so density is lagged by one iteration step in Eq. (1).

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To generate the finite-difference equations, central differencing is used. For example, $\rho A_1\phi_\xi$ becomes

$$\frac{1}{\Delta\xi}\left[(\rho A_1)_{i+\frac{1}{2}j}\left(\frac{\phi_{i+j}-\phi_{ij}}{\Delta\xi}\right)-(\rho A_1)_{i-\frac{1}{2}j}\left(\frac{\phi_i-\phi_{i-1j}}{\Delta\xi}\right)\right] \tag{3}$$

The quantities at half-points, such as $(\rho A_1)_{i+\frac{1}{2}j}$, are evaluated by simple averaging:

$$(\rho A_1)_{i+\frac{1}{2}j}=\tfrac{1}{2}[(\rho A_1)_{i+1j}+(\rho A_1)_{ij}] \tag{4}$$

The use of central difference is accurate and stable in subsonic regions. For supersonic regions, quantities are influenced by upstream points only and central differencing is no longer appropriate. Instead, an upwind bias is applied. This is simply and efficiently done by modifying the density to ρ, where

$$\begin{aligned}\hat\rho&=\rho_{ij} && \text{for } M_{\text{local}}<1\\ \hat\rho&=\rho_{ij}+\mu(\rho_{i-1j}-\rho_{ij}) && \text{for } M_{\text{local}}>1\end{aligned} \tag{5}$$

where $\mu=2(M_{\text{local}}^2-1)$ is an empirical factor. If the local Mach number is such that $\mu>1$, then μ is set equal to unity.

The cross derivatives and the density terms are evaluated at the old iteration level. Then, since the metric quantities A_1, A_2, and A_3 are given from the grid, Eq. (1) can be recast in the form

$$BP_{ij-1}^{n+1}+DP_{i-1j}^{n+1}+EP_{ij}^{n+1}+FP_{i+1j}^{n+1}+HP_{ij+1}^{n+1}=-R^n \tag{6}$$

where

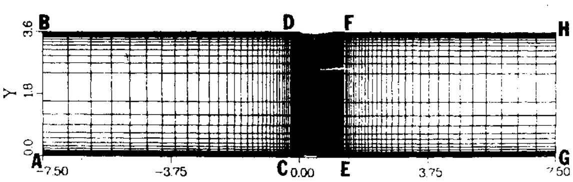
$$P_{ij}^{n+1}=\phi_{ij}^{n+1}-\phi_{ij}^n \tag{7}$$

and

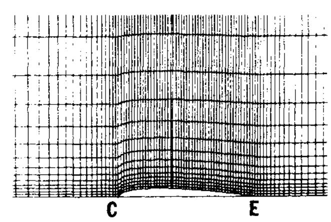
$$R^n=[(\rho A_1\phi_\xi+\rho A_2\phi_\eta)_\xi+(\rho A_2\phi_\xi+\rho A_3\phi_\eta)_\eta]^n \tag{8}$$

Equation (6) is solved using the SIP procedure of Ref. 7. The density is then solved by using Eq. (2) with central differences as needed.

Due to its simplicity, a sheared Cartesian grid, as shown in Fig. 1, was used for the coordinate mesh. Along the inflow



a)



b)

Fig. 1 a) Sheared Cartesian grid; b) detail in vicinity of airfoil.

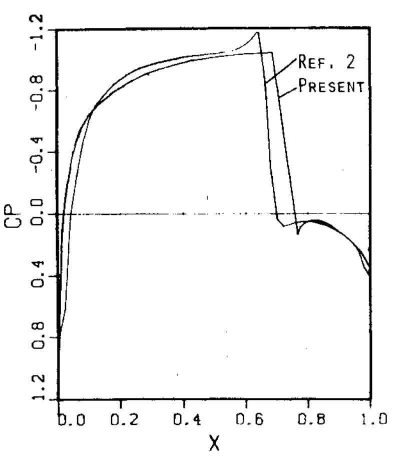


Fig. 2 Computed C_p distribution for NACA 0012 airfoil cascade; $M_\infty = 0.8$, $\beta_{in} = \beta_{out} = \alpha_{stagger} = 0$, solidity – 3.6.

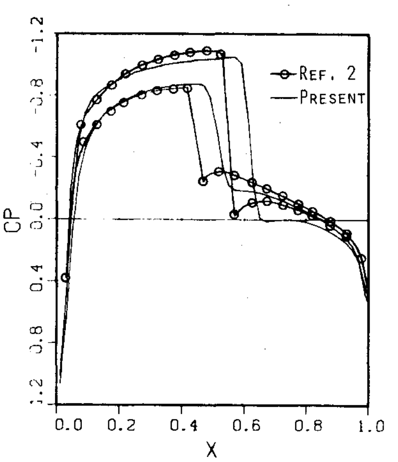


Fig. 3 Computed C_p distribution for NACA 0012 airfoil cascade; $M_\infty = 0.78$, $\beta_{in} = 0$, β_{out} determined from Kutta condition, $\alpha_{stagger} = 1$, solidity = 3.6.

boundary AB the values of ρ and ϕ were held fixed. In this case $\rho = 1$ and $\phi = x\cos\beta_{in} + y\sin\beta_{in}$, ($\beta_{in}$ is the inflow angle).

The outflow was assumed far enough away so that the flow is uniform, though inclined at β_{out}. The density is set to unity and ϕ calculated by linear interpolation between the value in the wake and upper boundary. Hence

$$\phi_{Ij} = \phi_{II} + \frac{y_j - y_I}{y_{j\max} - y_I} (\phi_{Ij\max} - \phi_{II}) \tag{9}$$

The two sets of periodic boundaries are denoted as the BD,AC and FH,EG pairs in Fig. 1. On the upstream edge, two conditions determine ϕ. The first is that the vertical component of the velocity is continuous, i.e.,

$$v_{ij\max} = v_{il} \quad \text{on BD and AC}$$

or

$$(\phi_\xi \xi_y + \phi_\eta \eta_y)_{ij\max} = (\phi_\xi \xi_y + \phi_\eta \eta_y)_{il} \tag{10}$$

and the second is that the jump in ϕ is provided by the inflow conditions so that

$$\phi_{ij\max} - \phi_{il} = h\cos\beta_{in} \quad \text{on BD and AC} \tag{11}$$

For the other pair, FH and EG, v is again to be continuous, but here the jump in ϕ is determined by the circulation or difference in ϕ at the two trailing edges. Hence

$$\phi_{ij\max} - \phi_{il} = \phi_{i_{TE}j\max} - \phi_{i_{TE}l} \quad \text{on FH and EG} \tag{12}$$

For the solid boundaries, the zero normal velocity condition $\phi_n = 0$ is applied. This gives

$$\phi_\xi (\xi_x \eta_x + \xi_y \eta_y) + \phi_\eta (\eta_x^2 + \eta_y^2) = 0 \tag{13}$$

ϕ_ξ is evaluated on the body at the old time level and

$$\phi_\eta = -\tfrac{1}{2} (3\phi_{ij}^{n+1} - 4\phi_{ij+1}^{n+1} + \phi_{ij+2}^{n+1}) \tag{14}$$

which includes the unknown value at the wall. Solving for the unknown ϕ with Eq. (13) gives

$$\phi_{ij} = \tfrac{1}{3} [4\phi_{ij+1} - \phi_{ij+2} - 2(A_2/A_3)\phi_\xi^n] \tag{15}$$

The density of the solid boundary is evaluated using Eq. (2) and the one-sided Eq. (14) for the η derivative at the wall.

Results

The pressure coefficient distributions for the cascaded airfoils are given in Figs. 2 and 3. Figure 2 represents the case of $M_\infty = 0.8$, zero stagger angle and flow aligned with the airfoil. In Fig. 3 the stagger angle is 1 deg, $\beta_{in} = 0$ deg, and $M_\infty = 0.78$. These results are compared with the finite-element calculations of Ecer and Akay.[2]

The results agree favorably, although there is a 5-6% discrepancy in the shock location. Further, the "rise" in C_p near the leading edge in Fig. 2 does not agree very well with Ecer and Akay. Our calculations here were done on the sheared Cartesian grid and probably account for the difference at the leading edge. Ecer and Akay also use a sheared grid but modify the leading edge region to reduce the skewness there. The shock location discrepancy in Fig. 3 is probably accounted for by the different treatment of boundaries downstream of the airfoil. Ecer and Akay specify the exit flow angle, which overspecifies the problem. Among other things, this means that the pressure at the trailing edge might not be smooth, although if the downstream boundary is far enough away the effect is lessened. In our calculations, the Kutta condition is satisfied across the wake, and the flow exit angle is part of the solution. Another possibility is that the finite element procedure multiplies the conservation equations by weighting functions before the Galerkin integration by parts. It thus becomes uncertain as to whether strict conservation, which is important for shock resolution and location, is maintained.

The computations were performed using a 151×30 grid on a VAX 11/780 machine. Typically 360 iterations were used. A similar version of the code has been run on a CDC 7600 machine and consumed 0.1-s CPU time per iteration, which compares favorably with Holsts' AF2 code.[5]

Conclusions

A general computer program has been developed to treat transonic potential flow past arbitrary cascade geometries. Satisfactory agreement has been observed between results computed by this program and published results.

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